

Heating up Galilean holography

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ABSTRACT: We embed a holographic description of a quantum field theory with Galilean conformal invariance in string theory. The key observation is that such field theories may be realized as conventional superconformal field theories with a known string theory embedding, twisted by the R-symmetry in a light-like direction. Using the Null Melvin Twist, we construct the appropriate dual geometry and its non-extremal generalization. From the nonzero temperature solution we determine the equation of state. We also discuss the hydrodynamic regime of these non-relativistic plasmas and show that the shear viscosity to entropy density ratio takes the universal value $\eta/s = 1/4\pi$ typical of strongly interacting field theories with gravity duals.

KEYWORDS: AdS-CFT Correspondence, Black Holes in String Theory.

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1. Introduction

The AdS/CFT correspondence [1–3] maps relativistic conformal field theories holographically to gravitational (or stringy) dynamics in a higher dimensional asymptotically Anti-de Sitter spacetime. As such, the correspondence is an important tool for modeling the behavior of strongly interacting field theories, as the dynamics on the field theory side is mapped to classical string and gravitational dynamics in the dual description. Indeed, among other achievements, this strong-weak coupling duality has improved, at least at a qualitative level, our understanding of real-time dynamics and transport properties of the quark-gluon plasma in QCD.

More recently, these holographic ideas have been applied to conformal field theories arising from condensed matter systems. There is a large class of interesting strongly correlated electron and atomic systems that can be created and studied in table-top experiments. In special cases, these systems exhibit relativistic dispersion relations, and the dynamics near a critical point is then well described by a relativistic conformal field theory. It

is precisely such field theories which may be studied using holographically dual AdS geometries. Recent work (see refs. [4] for a sampling) has already applied this AdS/CMat correspondence to strongly correlated electrons, superconductors, the quantum Hall effect, and more.

More ambitiously, one can ask whether the holographic approach can be extended to the non-relativistic theories describing most condensed matter systems. In particular, can field theories with Galilean scaling symmetry (see refs. [5–7] for discussions of non-relativistic CFTs) have a holographic dual? Just as the Poincaré algebra can be extended to the conformal algebra in relativistic quantum field theories, one can extend the Galilean algebra (symmetry of non-relativistic field theories) to the so called Schrödinger algebra [5]. Fermions at unitarity are conjectured to realize this Schrödinger symmetry. The scale invariance is achieved by fine tuning the fermions — with an external magnetic field for example — to obtain a massless bound state, thus making the scattering length effectively infinite. These systems are of increasing interest in the context of trapped cold atoms at a Feshbach resonance. Indeed refs. [8–11] claim that these cold atom systems provide another example of a nearly ideal fluid with very low viscosity, like the quark-gluon plasma. See refs. [12–16] for experimental studies of cold atom systems and their hydrodynamic transport coefficients. Indeed the latest result of ref. [16] predicts a value of η/s slightly above the values obtained for the quark-gluon plasma in heavy-ion collisions, putting it well into the strongly coupled regime. Having a holographic description would certainly be helpful for understanding the strongly coupled dynamics one encounters in these systems.

Important steps in this direction were taken in refs. [17, 18], where gravitational backgrounds dual to non-relativistic conformal field theories were proposed.¹ These dual geometries involve a pp-wave deformation of AdS. In this paper, we will continue the exploration of the bulk geometry proposed as the dual of non-relativistic conformal field theories. We will show how the geometry can be realized in a string theory context, and discuss non-extremal generalizations, dual to non-relativistic conformal field theories at nonzero temperature. A slightly different duality involving pure AdS bulk spacetimes was proposed in refs. [20, 21]. In the next section, we will review the proposed duality of refs. [17, 18], and consider the advantages and disadvantages compared to the rival proposals of refs. [20, 21].²

In section 3 we will show how the geometries of refs. [17, 18] can arise in string theory, and construct nonzero-temperature generalizations of them. We describe how the solutions can be constructed by the Null Melvin Twist [23, 24], which was originally invented to construct asymptotically plane wave black holes. For the case of $d = 2$ spatial dimensions the conformally invariant theory will be realized as the world-volume theory on D3-branes with a light-like twist in the R-symmetry directions. Starting from $\mathcal{N} = 4$ Super-Yang Mills (SYM) on the D3-brane world volume, the effect of the twist can be understood as adding a dimension five Lorentz violating operator, which deforms the asymptotic AdS geometry to the desired pp-wave form. The twist will break the R-symmetry of $\mathcal{N} = 4$ SYM from $SU(4)$ down to $SU(3) \times U(1)$.³ We thus realize a non-relativistic conformal field

¹See also refs. [19] for related work.

²See also ref. [22] for related work.

³There are twists of the R-symmetry that preserve as many as 8 supercharges [23]. We use a simple

theory directly in terms of a discrete light-cone quantization (DLCQ) of a deformation of $\mathcal{N} = 4$ SYM. Such theories were discussed previously in [25, 23] and belong to a class of non-local field theories called dipole theories.⁴

Another interesting class of non-relativistic field theories which arise in string theory are a special case of non-commutative field theories; in these theories the geometry is supported by fluxes which break the spatial rotational symmetries. These do not have the Schrödinger symmetry but obey a generalized scaling symmetry. The simplest such model is the light-like non-commutative $\mathcal{N} = 4$ SYM described originally in ref. [26], whose holographic dual was constructed in ref. [27]. These geometries were investigated for their causality properties in refs. [28, 29], where the Galilean structures were naturally shown to arise; we will discuss this issue of causal structure further in section 2 and some specific examples in appendix A.

As the spacetime geometries we consider are constructed by the Null Melvin Twist solution generating technique, it is easy to construct the non-extremal versions of the solutions considered in refs. [17, 18]. In section 4, we give a preliminary consideration of the thermodynamics of the non-extremal geometries. We argue that the black hole solution which is dual to the thermal version of the twisted D3-brane theory corresponds to working in a grand canonical ensemble with chemical potential for the particle number (realized as momentum in the light-cone direction). Given this interpretation, we discuss how to obtain this grand canonical partition function via a Euclidean quantum gravity saddle point computation. We then undertake a detailed investigation of the asymptotics and action for the spacetimes of interest in section 5, recovering from this analysis the complete thermodynamics of the dual non-relativistic field theory. We then turn to the hydrodynamic description of the non-relativistic conformal plasmas and calculate the shear-viscosity in section 6. We find that η/s has the universal value $1/4\pi$ typical of strongly interacting field theories with gravity duals [30]. We conclude with a discussion in section 7. In appendix A we discuss how to obtain metrics of the form (2.1) supported by p -form fluxes that break rotational invariance and their realization in string theory.

Note added: in concurrent work Adams, Balasubramanian, and McGreevy [31] and Maldacena, Martelli and Tachikawa [32] have obtained very similar results to ours. We would like to thank Allan Adams for extensive discussions regarding their results during the BIRS workshop.

2. The geometry dual to Galilean CFTs

We begin with a brief review of the proposed holographic duality for non-relativistic field theories of refs. [17, 18, 20, 21]. In these proposals, the non-relativistic conformal symmetry is realized as a subset of a relativistic conformal symmetry with an additional dimension. The Schrödinger algebra is obtained from the relativistic conformal algebra by reducing

twist which does not preserve any supersymmetry because of the resulting form of the $H_{(3)}$ flux.

⁴We would like to thank Allan Adams for mentioning this DLCQ interpretation in informal discussion.

along a light-cone. The procedure is similar to light-cone quantization, where at fixed light-cone momentum only a Galilean subgroup of the Lorentz group is manifest.

The holographic dual of a d spatial dimensional Galilean CFT is then a gravitational solution in $d + 3$ dimensions. This dual spacetime should realize the Galilean scaling symmetry as an isometry. In ref. [17, 18], this spacetime is taken to have a metric of the form⁵

$$ds^2 = r^2 (-2 du dv - r^{2\nu} du^2 + d\mathbf{x}^2) + \frac{dr^2}{r^2}, \quad (2.1)$$

where $\mathbf{x} = \{x_1 \dots x_d\}$ are the spatial coordinates of the Galilean field theory. The light-cone coordinate u is the boundary time coordinate: the field theory Hamiltonian is conjugate to $\frac{\partial}{\partial u}$. The role of the v -direction is unclear; it is proposed that we treat this as a compact direction, in the spirit of DLCQ. As in AdS/CFT, the bulk coordinate r should correspond to scale size in the boundary field theory. The Galilean scaling symmetry is realized as

$$\mathbf{x} \sim \lambda \mathbf{x}, \quad u \sim \lambda^{\nu+1} u, \quad v \sim \lambda^{1-\nu} v, \quad r \sim \lambda^{-1} r. \quad (2.2)$$

In the special case of $\nu = 1$ it is expected that the scaling symmetry extends to full Galilean conformal invariance, realizing the Schrödinger algebra. In this special case, v is invariant under scaling; the Galilean scale invariance requires that the time coordinate, u , scales twice as fast as the spatial coordinates \mathbf{x} . For details of the Schrödinger algebra we refer the reader to ref. [7]. We will primarily focus on the $\nu = 1$ case, but will also mention realizations of the $\nu = 2$ case in terms of non-commutative field theories in appendix A.

The geometry (2.1) is a solution to Einstein's equations with negative cosmological constant, with matter whose stress tensor is of the null dust form $T_{uu} \propto r^{2\nu+2}$. Ref. [17] modeled the matter using a massive vector field, while ref. [18] used an Abelian-Higgs action. We will discuss below how to embed the construction of ref. [17] into Type IIB string theory and realize the line element (2.1) as the near-horizon geometry of a twisted D3-brane solution.

On the other hand, refs. [20, 21] proposed that the dual geometry can be pure AdS, with the relativistic conformal symmetry broken to Galilean symmetry simply by compactification of the v coordinate, which singles out a preferred light-cone direction. Such a modification of AdS would certainly be a simpler setting in which to study Galilean symmetry, but we feel that the geometry (2.1) is a more natural dual for such non-relativistic conformal theories.

The main reason we prefer the original proposal of refs. [17, 18] to the simplified proposal of refs. [20, 21] is that the causal structure of (2.1) naturally reproduces the Galilean light cone of the field theory. The causal structure of a non-relativistic field theory is degenerate — interactions can propagate instantaneously. While a bulk geometry with a well-behaved causal structure cannot be holographically dual to a non-relativistic field theory, the spacetime geometry (2.1) evades this issue beautifully — its causal structure is also degenerate and in such a way as to be consistent with the boundary Galilean invariance.⁶

⁵We consider here $\nu \neq 0$, since $\nu = 0$ is just AdS_{d+2} .

⁶The special case $\nu = 0$ is of course pure AdS with a well-behaved causal structure.

The spacetime (2.1) is conformal (with an overall conformal factor r^2) to a pp-wave spacetime, and this pp-wave spacetime is known to be non-distinguishing [33, 34]. Non-distinguishing means that while the spacetime (2.1) is causal (in the sense of not having closed causal curves), there are distinct points in the spacetime which have identical past and future sets,⁷ thereby preventing us from distinguishing spacetime events by reference to their past and future sets. In fact, in (2.1), all points on a surface with $u = u_0$ (and arbitrary values of other coordinates) have an identical causal future/past [34]. But Galilean CFTs have precisely such a causal structure; all spatial points on an equal time surface can influence any arbitrary spatial point at an infinitesimal time later.⁸ By contrast, a pure AdS spacetime with boundary conditions engineered to give Galilean invariance does not possess a bulk light-cone which agrees with the light-cone of the relativistic field theory.

This consistency of the bulk spacetime causal structure with the boundary causal structure is a crucial ingredient in the AdS/CFT correspondence. Without this agreement, we would easily be able to set up gedanken experiments where bulk physics and boundary physics would not agree. Consider for example the question of the singularity structure of the boundary correlation function as discussed in ref. [35]; in pure AdS the correlation functions will have a singular locus consistent with the boundary light-cone having full Lorentz invariance in one lower dimension. We however want a boundary light-cone consistent with a Galilean invariant field theory living in two lower dimensions, which the bulk correlators do not see unless we explicitly break boundary Lorentz invariance. On the contrary the geometries (2.1) will indeed give a singular locus of the correlators which is commensurate with a Galilean light-cone as discussed in ref. [28].

Another point in favour of the original proposal of refs. [17, 18] is that in the spacetime (2.1), the symmetry is broken to Galilean invariance irrespective of whether v is compact or not. For $\nu = 1$, compactification of v to obtain a DLCQ description doesn't break any further symmetry, so the period of compactification Δv is a physical parameter, which can be interpreted as the inverse of the Galilean mass in the non-relativistic CFT. On the other hand, if the spacetime is pure AdS, the symmetry is only broken to the Galilean invariance by compactification in v . Prior to compactification, we have boost invariance $u \rightarrow \lambda u$ and $v \rightarrow \lambda^{-1} v$ in addition to the scaling symmetry (2.2). This broken boost invariance can be used to relate different values of Δv , making the compactification radius an unphysical parameter. To be more explicit, we rewrite (2.1) in a more general form,

$$ds^2 = r^2 \left(-2 du dv - \beta^2 r^{2\nu} du^2 + d\mathbf{x}^2 \right) + \frac{dr^2}{r^2}. \quad (2.3)$$

Here we have introduced an additional parameter β : we can set $\beta = 1$ by a boost $u \rightarrow \beta u$, $v \rightarrow \beta^{-1} v$. If v is compact, the combination $\beta/\Delta v$ is invariant under this boost transformation. In the approach of refs. [17, 18], the boost is used to set $\beta = 1$, and the

⁷The timelike future $I^+(p)$ for a point p is the set of points which can be reached from p by future-directed timelike curves; timelike past is defined similarly. Causal future/past are defined likewise in terms of causal (timelike or null) curves.

⁸Note that refs. [28, 29], in considering the holographic dual of non-commutative $\mathcal{N} = 4$ Super-Yang Mills with light-like non-commutativity [27], have already studied the causal properties of precisely this $\nu = 2$ geometry.

invariant quantity $\beta/\Delta v$ is then interpreted as the Galilean mass. We can now recognize the pure AdS duality of refs. [20, 21] as the special case in which we set $\beta = 0$, so the boost invariant is zero, and the coordinate period Δv is not a physical parameter. From the non-relativistic CFT point of view, $\beta = 0$ is a limit in which the Galilean mass vanishes. This clarifies the observation in ref. [20] that the Galilean mass does not enter into the formula for operator dimensions in this simplified case. Pure AdS as a holographic dual is a degenerate special case of that of (2.1).

We will now proceed to embed (2.1) into string theory and realize a class of non-relativistic CFTs using conventional D-brane physics.

3. Embedding in string theory

The geometry (2.1) can be consistently embedded in a solution to string theory. Indeed, geometries of this type have previously been studied, in investigations of the application of solution generating transformations to construct geometries corresponding to twisted versions of the D3-brane worldvolume theory [27, 28]. In this section, we first review this solution generating transformation, and use it to construct a string theory solution which reduces to (2.1) in five dimensions. We then apply the same transformation to obtain a non-extremal generalization, and construct a five-dimensional theory for which the non-extremal geometry is a solution.

3.1 Generating the geometry dual to the vacuum state

To begin with, consider the geometry of $\text{AdS}_5 \times \mathbf{S}^5$ in Poincaré coordinates, which is the near-horizon geometry of D3-branes in flat space:

$$\begin{aligned}
 ds^2 &= r^2 (-dt^2 + d\mathbf{x}^2 + dy^2) + \frac{dr^2}{r^2} + (d\psi + A)^2 + d\Sigma_4^2, \\
 F_{(5)} &= dC_{(4)} = 2(1 + \star) d\psi \wedge J \wedge J,
 \end{aligned}
 \tag{3.1}$$

where we have written the metric on the unit \mathbf{S}^5 as a fibration over a \mathbf{CP}^2 base and now $\mathbf{x} = \{x_1, x_2\}$. The five-form is given explicitly in terms of the volume form of \mathbf{S}^5 , which has been decomposed into quantities related to the fibration. J is the Kähler form on \mathbf{CP}^2 and A is the associated potential. Our conventions are

$$dA = 2J, \quad \text{Vol}(\mathbf{CP}^2) = \frac{1}{2} J \wedge J.
 \tag{3.2}$$

We apply a Null Melvin Twist to this geometry, as described in [24]; the idea is to generate light-like NS-NS flux by a series of boosts and twisted T-dualities. Algorithmically we proceed as follows:⁹

1. Pick a translationally invariant direction (say y) and boost by amount γ along y .
2. T-dualize along y .

⁹The D3-brane geometry above has a full $\text{SO}(1,1)$ symmetry in the (t, y) plane which renders the first step inconsequential here, but it will be meaningful for the non-extremal solution which follows.

3. Twist some one-form σ : $\sigma \rightarrow \sigma + \alpha dy$.
4. T-dualize along y again.
5. Boost by $-\gamma$ along y .
6. Scale the boost and twist: $\gamma \rightarrow \infty$ and $\alpha \rightarrow 0$, keeping

$$\beta = \frac{1}{2} \alpha e^\gamma = \text{fixed.} \tag{3.3}$$

The only data needed to describe the construction is the choice of the one-form σ . We can choose σ to be along the world-volume directions (linear combination of dx_1 and dx_2) or transverse to the D-brane. The former leads to turning on constant electric and magnetic fields on the D-brane world-volume leading to a light-like non-commutative field theory [27, 28]. Due to the presence of world-volume fluxes these geometries break the rotational invariance in the \mathbf{x} directions and they also give rise to geometries (2.1) with $\nu \neq 1$; we will not consider them in detail in the main text of the paper, but discuss aspects of these geometries in appendix A.

Twisting along the R -symmetry direction is more interesting. A natural choice is to take the one-form σ to be along the fiber direction: $\sigma = d\psi$. The Null Melvin Twist leads to the geometry [28]

$$\begin{aligned} ds^2 &= r^2 (-2 du dv - r^2 du^2 + d\mathbf{x}^2) + \frac{dr^2}{r^2} + (d\psi + A)^2 + d\Sigma_4^2, \\ F_{(5)} &= 2(1 + \star) d\psi \wedge J \wedge J, \\ B_{(2)} &= r^2 du \wedge (d\psi + A), \end{aligned} \tag{3.4}$$

where the light-cone coordinates are

$$u = \beta(t + y), \quad v = \frac{1}{2\beta}(t - y). \tag{3.5}$$

Note that our boosted uv coordinate frame scales β out not only from the metric but also from the field strengths. The five-dimensional part of this metric is precisely the geometry (2.1), with $\nu = 1$ and $d = 2$. This geometry will correspond to the vacuum state of the dual non-relativistic field theory.

The Null Melvin Twist construction makes the interpretation of the dual field theory clean: it is nothing but $\mathcal{N} = 4$ Super Yang-Mills twisted by an R-charge. The $U(1)$ isometry generating the R-charge is generated in the spacetime by $\frac{\partial}{\partial \psi}$. This twist breaks the $SU(4)$ symmetry of $\mathcal{N} = 4$ down to an $SU(3) \times U(1)$ (the isometry group of \mathbf{CP}^2) through the non-vanishing NS-NS potential $B_{(2)}$ (the metric (3.4) of course enjoys full $SU(4)$ invariance).

3.2 The non-extremal solutions

As we have generated (3.4) by a solution generating technique, we can just as well generate the non-extremal version of the solution. To do so, rather than starting with the near

horizon geometry of extremal D3-branes, we start with non-extremal D3-branes and repeat the Null Melvin Twist. Consider then the planar Schwarzschild-AdS black hole (times \mathbf{S}^5 , with the geometry supported by the five-form flux $F_{(5)}$)

$$ds^2 = r^2 (-f(r) dt^2 + dy^2 + d\mathbf{x}^2) + \frac{1}{r^2} \left(\frac{dr^2}{f(r)} + r^2 d\Omega_5^2 \right), \quad (3.6)$$

where as before we will write the \mathbf{S}^5 as a \mathbf{S}^1 fibration over \mathbf{CP}^2 . The Null Melvin Twist leads to the string frame metric [36]:

$$\begin{aligned}
 ds_{\text{str}}^2 &= r^2 \left(-\frac{\beta^2 r^2 f(r)}{k(r)} (dt + dy)^2 - \frac{f(r)}{k(r)} dt^2 + \frac{dy^2}{k(r)} + d\mathbf{x}^2 \right) \\
 &\quad + \frac{dr^2}{r^2 f(r)} + \frac{(d\psi + A)^2}{k(r)} + d\Sigma_4^2, \\
 e^\varphi &= \frac{1}{\sqrt{k(r)}}, \\
 F_{(5)} &= dC_{(4)} = 2(1 + \star) d\psi \wedge J \wedge J, \\
 B_{(2)} &= \frac{r^2 \beta}{k(r)} (f(r) dt + dy) \wedge (d\psi + A),
 \end{aligned} \quad (3.7)$$

with

$$f(r) = 1 - \frac{r_+^4}{r^4}, \quad k(r) = 1 + \beta^2 r^2 (1 - f(r)) = 1 + \frac{\beta^2 r_+^4}{r^2}. \quad (3.8)$$

The solution has a horizon at $r = r_+$. Note that the parameter β appearing in this metric is an independent physical parameter; in the extremal case, we could set it to one by boosting in the ty plane, but non-extremality has broken this boost symmetry. The remainder of the paper will be devoted to an exploration of the physics of this non-extremal solution. First, in the next section, we construct an appropriate five-dimensional Lagrangian which has (3.7) as a solution.

3.3 Five dimensional effective lagrangian

The solutions we have discussed above (3.4) and (3.7) satisfy the 10-dimensional Type IIB equations of motion. In [17], the vacuum geometry (2.1) was considered as a solution to Einstein-Proca theory with negative cosmological constant, which has the action

$$\mathcal{S}_{\text{EP}} = \int d^{d+2}x dr \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m^2 A^\mu A_\mu \right), \quad (3.9)$$

with $F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$. The metric (2.1) with $A^v = 1$ satisfies the field equations for the choice

$$\Lambda = -\frac{1}{2} (d+1)(d+2), \quad m^2 = 2(d+2). \quad (3.10)$$

We would now like to understand the relation between this phenomenological Lagrangian and the ten-dimensional IIB theory. Starting from Type IIB supergravity, we can KK reduce the solution (3.4) on the \mathbf{S}^5 (which is undeformed). The reduction of the metric is straightforward, and gives (2.1) in five dimensions. The NS-NS two-form, however, depends on the \mathbf{S}^5 coordinates. In a linear analysis [37], such a mode of the two-form

produces a massive vector transforming in the **15** of $\text{SO}(6)$: in AdS units (set here to 1) its mass is $m^2 = 8$. This is precisely the value of the mass required according to (3.10) (with $\Lambda = -6$ as necessary to get AdS radius equal to 1).

From the CFT point of view, this massive vector field in the bulk corresponds to a dimension 5 operator in $\mathcal{N} = 4$ SYM. The twist by R-symmetry is by an irrelevant operator of dimension 5 transforming in the antisymmetric tensor representation of $\text{SU}(4)$. The operator in question [23] is $\mathcal{O}_\mu^{IJ} = \text{Tr} \left(F_\mu^\nu \Phi^{[I} D_\nu \Phi^{J]} + \sum_K D_\mu \Phi^K \Phi^{[K} \Phi^I \Phi^{J]} \right) + \text{fermions}$, where Φ^I are the adjoint scalars of $\mathcal{N} = 4$ SYM transforming in the vector **6** of $\text{SU}(4)$ and $F_{\mu\nu}$ is the gauge field strength. The Lorentz symmetry is broken by adding \mathcal{O}_u^{IJ} to the field theory Lagrangian. This field theory realization makes it clear that the massive vector used in the construction of [17] oxidises to NS-NS flux in ten dimensions.

It is, however, important to note that this massive vector is not part of gauged supergravity in five dimensions. Thus, it is not obvious that (3.9) is a consistent truncation of the ten-dimensional theory. That is, while we have found an embedding of (2.1) in the ten-dimensional geometry (3.4), we have no guarantee that solutions of (3.9) can in general be oxidised to solutions of the ten-dimensional IIB equations of motion.

If we perform the same Kaluza-Klein reduction for the non-extremal solution (3.7), we obtain

$$\begin{aligned}
 ds_E^2 &= r^2 k(r)^{-\frac{2}{3}} \left(-\beta^2 r^2 f(r) (dt + dy)^2 - f dt^2 + dy^2 + k d\mathbf{x}^2 \right) + k(r)^{\frac{1}{3}} \frac{dr^2}{r^2 f(r)}, \\
 &= r^2 k(r)^{-\frac{2}{3}} \left(\left[\frac{1-f(r)}{4\beta^2} - r^2 f(r) \right] du^2 + \frac{\beta^2 r_+^4}{r^4} dv^2 - [1+f(r)] du dv \right) \\
 &\quad + k(r)^{\frac{1}{3}} \left(r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2 f(r)} \right), \tag{3.11}
 \end{aligned}$$

where we have introduced the light-cone coordinates (3.5) in the second line for future convenience, with the massive vector and scalar

$$\begin{aligned}
 A &= \frac{r^2 \beta}{k(r)} (f(r) dt + dy) = \frac{r^2}{k(r)} \left(\frac{1+f(r)}{2} du - \frac{\beta^2 r_+^4}{r^4} dv \right), \\
 e^\phi &= \frac{1}{\sqrt{k(r)}}, \tag{3.12}
 \end{aligned}$$

where $f(r)$ and $k(r)$ are given in (3.8). Note that in these light-cone coordinates, the solution asymptotically approaches the extremal solution (2.1), but β remains a physical parameter, as the full metric depends on β . We will henceforth work with the solution (3.11).

This black hole solution is not a solution of (3.9), as it contains a scalar field. However, it is a solution of the equations of motion from the effective 5 dimensional action

$$\mathcal{S} = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{1}{4} e^{-8\phi/3} F_{\mu\nu} F^{\mu\nu} - 4 A_\mu A^\mu - V(\phi) \right), \tag{3.13}$$

where the scalar potential is

$$V(\phi) = 4 e^{2\phi/3} (e^{2\phi} - 4), \tag{3.14}$$

The scalar here appears from two sources: (i) the black hole geometry involves a non-vanishing dilaton and (ii) the twist now causes the fibration over \mathbf{CP}^2 to be squashed. Squashing is a common feature of solutions generated by the Null Melvin Twist [24] and intuitively can be ascribed to the distortion of the asymptotics of the spacetime.

To summarize, it is easy to embed geometries of the form (2.1) into string theory. The dimensionally reduced descriptions appear to involve exotic matter like Proca fields as in (3.13), but their stringy origin is simply in terms of conventional supergravity p -forms.

Again, we do not have any argument that the five-dimensional action (3.13) describes a consistent truncation of the full ten-dimensional theory, and indeed, one might in general expect the modes transforming non-trivially under $SO(6)$ which are turned on in our ansatz to couple to other Kaluza-Klein harmonics which we have neglected. However, by construction we know this particular five-dimensional solution (3.11) uplifts to type IIB supergravity, and from now on, we will work with the five-dimensional action (3.13). In appendix A, we discuss a different way of realizing geometries like (2.1), using p -form fluxes which break the rotational invariance in the spatial directions. This alternative approach can be easily embedded in five-dimensional gauged supergravity, using a Null Melvin Twist along the worldvolume directions.

4. Thermodynamics from gravity

We are interested in understanding the thermodynamics of the black hole solution (3.11). The simplest thing to understand is the entropy of the black hole. The geometry (3.11) has a horizon at $r = r_+$ and one can compute the area of this horizon. In fact, since we generated the solution by a series of boosts and dualities, it turns out that the horizon area is independent of β [24]. We obtain thus

$$S = \frac{1}{4G_5} r_+^3 \text{Vol}(horizon) = \frac{1}{4G_5} r_+^3 \Delta y \Delta x_1 \Delta x_2. \tag{4.1}$$

We can also compute the Hawking temperature of the black hole, which is most simply done by computing the surface gravity of the horizon. Given the null generator of the horizon, ξ^a , the surface gravity κ is defined as

$$\kappa^2 = -\frac{1}{2} \left(\nabla^a \xi^b \right) \left(\nabla_a \xi_b \right), \tag{4.2}$$

with the evaluation at the location of the horizon implicit. The Hawking temperature is given in terms of the surface gravity as $T_H = \frac{1}{2\pi} \kappa$.

It is clear from (3.11) that the Killing generator of the event horizon is proportional to $\frac{\partial}{\partial t}$. To fix the constant of proportionality, we require that the component along the boundary time-translation have coefficient one. The generator of time translation for our non-relativistic CFT is $\frac{\partial}{\partial u}$ in the light cone coordinates (3.5), so this requirement fixes the

normalization of the Killing generator of the horizon as

$$\xi = \frac{1}{\beta} \frac{\partial}{\partial t} = \frac{\partial}{\partial u} + \frac{1}{2\beta^2} \frac{\partial}{\partial v}. \tag{4.3}$$

With this definition we find that

$$T = \frac{r_+}{\pi \beta}. \tag{4.4}$$

The Killing generator of the event horizon (4.3) not only has components along the boundary time translation direction u , but also along the light-like direction v . From the gravitational perspective it therefore appears that we are dealing with a system where we have a chemical potential for v -translations

$$\mu = \frac{1}{2\beta^2}. \tag{4.5}$$

The conserved charge conjugate to this chemical potential is just $\frac{\partial}{\partial v}$ momentum. We therefore claim that the black hole solution (3.11) corresponds to thermodynamics described by the density matrix for a grand canonical ensemble

$$\rho = \exp\left(-\frac{\hat{H}}{T} - \frac{\mu \hat{P}_v}{T}\right). \tag{4.6}$$

In the dual field theory, the operator generating translations in u is the Hamiltonian \hat{H} , while the v momentum corresponds to the integer quantized particle number \hat{N} , $\hat{P}_v = 2\pi\hat{N}/\Delta v$. Note that the operator \hat{N} commutes with the Hamiltonian \hat{H} . In fact the only place it shows up in the Galilean conformal algebra [7] is in the commutator of spatial momentum and Galilean boosts, $[\hat{P}^i, \hat{K}^j] = -i\delta^{ij} \hat{N}$, as it commutes with all the other generators of the Schrödinger algebra. The Gibbs potential of this ensemble $\mathcal{Q}(T, \mu, V)$ can be calculated from the partition function

$$\Xi(T, \mu) = \text{Tr}(\rho), \quad \mathcal{Q}(T, \mu, V) = -T \log \Xi(T, \mu). \tag{4.7}$$

We can then pass to a more conventional ensemble with fixed particle number N by a Legendre transformation leading to a free energy $F(T, N, V)$ in the canonical ensemble

$$F(T, N, V) = \mathcal{Q}(T, \mu, V) - \mu N. \tag{4.8}$$

The non-relativistic theories we are dealing with are realized as a deformed version of a relativistic quantum field theory. Hence it is not surprising that the natural ensemble is one where the particle number is allowed to fluctuate. Another argument for the naturalness of the grand canonical ensemble comes from the geometry. The Galilean theories we constructed are embedded into a higher dimensional Poincaré invariant theory (in the present case $\mathcal{N} = 4$ SYM), in which we turn on some background fields to break Poincaré invariance. We can reduce the theory on the light-cone to obtain a Galilean CFT, but in this procedure there is no canonical choice of the P_v eigenvalue $2\pi N/\Delta v$. It therefore seems natural to sample over the space of eigenvalues weighted by some parameter μ .

5. Asymptotics and action

We now turn to the calculation of the Gibbs potential $\mathcal{Q}(T, \mu, V)$ in a saddle-point approximation. This potential can be obtained from the on-shell action of an analytically continued version of the black hole solution. The geometry (3.11) does not have a real Euclidean section because of the presence of non-zero chemical potentials, but we can still use the analytically continued metric in a saddle-point approximation: as argued in [38] (in the context of the Kerr-Newman solutions), the appropriate saddle point for the thermodynamic partition function is obtained by analytic continuation of the time coordinate, even in cases where the resulting metric is complex. The action evaluated on this analytically continued solution is always real, so it can be used as a saddle-point approximation to the thermodynamic potential.

We would also like to be able to derive a boundary stress tensor using an extension of the Brown-York type analysis used in the context of AdS/CFT [39, 40]. It is not clear that this technique can be straightforwardly applied to our solution, because of the inhomogeneity in the asymptotic falloff conditions for different components of the metric, which implies that we do not have a regular conformal structure on the boundary.¹⁰ However, a first step in such a calculation is the determination of the necessary boundary terms required to obtain a well-behaved action. A well defined action should have vanishing variation on-shell in a classical phase space that encompasses the solutions we are interested in.

We will start by analyzing the asymptotic fall-off conditions for the black hole space-time. We then proceed to construct an action that is stationary with respect to an appropriate class of variations. Our main result is summarized in (5.11), which we then use to extract the Gibbs potential $\mathcal{Q}(T, \mu, V)$ from a saddle point evaluation.

5.1 Naive asymptotics from metric expansion

To understand the asymptotics, let us focus on the five dimensional solution (3.11) in the light-cone coordinates (3.5). We find

$$\begin{aligned}
 g_{uu} &= -r^4 + \frac{2}{3}\gamma^2 r^2 + \mathcal{O}(1), & g_{uv} &= -r^2 + \frac{2}{3}\gamma^2 + \mathcal{O}(r^{-2}), \\
 g_{vv} &= \frac{\gamma^2}{r^2} + \mathcal{O}(r^{-4}), & g_{xx} &= r^2 + \frac{1}{3}\gamma^2 + \mathcal{O}(r^{-2}), \\
 g_{rr} &= \frac{1}{r^2} + \frac{1}{3}\frac{\gamma^2}{r^4} + \mathcal{O}(r^{-6}),
 \end{aligned}
 \tag{5.1}$$

and for the inverse metric,

$$\begin{aligned}
 g^{uu} &= -\frac{\gamma^2}{r^6} + \mathcal{O}(r^{-8}), & g^{uv} &= -\frac{1}{r^2} + \frac{1}{3}\frac{\gamma^2}{r^4} + \mathcal{O}(r^{-6}), \\
 g^{vv} &= 1 - \frac{1}{3}\frac{\gamma^2}{r^2} + \mathcal{O}(r^{-4}), & g^{xx} &= \frac{1}{r^2} - \frac{1}{3}\frac{\gamma^2}{r^4} + \mathcal{O}(r^{-6}), \\
 g^{rr} &= r^2 - \frac{1}{3}\gamma^2 + \mathcal{O}(r^{-2}),
 \end{aligned}
 \tag{5.2}$$

¹⁰From the discussion of the causal properties in section 2 it follows that we also cannot define a causal boundary for (2.1).

where for ease of notation, we introduce $\gamma^2 \equiv \beta^2 r_+^4$, which encodes the leading deformation of the vacuum spacetime (2.1). For the matter fields it follows from (3.12) that

$$\begin{aligned} A_u &= r^2 - \gamma^2 + \mathcal{O}(r^{-2}), & A_v &= -\frac{\gamma^2}{r^2} + \mathcal{O}(r^{-4}), \\ \phi &= -\frac{\gamma^2}{2r^2} + \mathcal{O}(r^{-4}). \end{aligned} \tag{5.3}$$

It is worth remarking that these fall-off conditions are a result of a coupling between the linearized fluctuations of the fields about the vacuum background (3.4); the fluctuations of the gravitational, vector and scalar degrees of freedom do not decouple in the five-dimensional effective action (3.13).

5.2 A stationary action

We want to evaluate the on-shell action for the solution (3.11). To get a well-behaved action, we need to supplement the bulk action (3.13) with boundary terms to satisfy the condition that $\delta\mathcal{S} = 0$ on-shell, for variations satisfying suitable falloff conditions. In this subsection, we construct an action satisfying this condition for a very restricted set of variations — the minimal set including variations along the family of solutions we're interested in.

We will construct an action principle by adding local covariant counterterms to the bulk action plus a Gibbons-Hawking boundary term, as in asymptotically AdS spacetimes [39, 40]. If we add the most general combination of local counterterms which can make non-zero contributions to the on-shell action, we have

$$\begin{aligned} \mathcal{S} &= \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{4}e^{-8\phi/3} F_{\mu\nu}F^{\mu\nu} - 4A_\mu A^\mu - V(\phi) \right) \\ &\quad + \frac{1}{16\pi G_5} \int d^4\xi \sqrt{-h} \left(2K - 2c_0 + c_1\phi + c_2\phi^2 \right. \\ &\quad \left. + c_3 A_\alpha A^\alpha + c_4 A_\alpha A^\alpha \phi + c_5 (A_\alpha A^\alpha)^2 \right). \end{aligned} \tag{5.4}$$

Here ξ^α are boundary coordinates, $h_{\alpha\beta}$ is the induced metric on the boundary, and $K = K_{\alpha\beta}h^{\alpha\beta}$, with $K_{\alpha\beta}$ the extrinsic curvature. We want to fix the coefficients c_0, c_1, c_2, c_3, c_4 by imposing $\delta\mathcal{S} = 0$. In general, the variation of this action is

$$\begin{aligned} \delta\mathcal{S} &= \frac{1}{16\pi G_5} \int d^4\xi \sqrt{-h} \left[\left(\pi_{\alpha\beta} + \left(c_0 - \frac{1}{2}\phi c_1 - \frac{1}{2}\phi^2 c_2 \right) h_{\alpha\beta} \right. \right. \\ &\quad \left. \left. + \left(A_\alpha A_\beta - \frac{1}{2} A_\gamma A^\gamma h_{\alpha\beta} \right) (c_3 + \phi c_4) \right. \right. \\ &\quad \left. \left. + \left(2A_\alpha A_\beta - \frac{1}{2} A_\gamma A^\gamma h_{\alpha\beta} \right) c_5 (A_\delta A^\delta) \right) \delta h^{\alpha\beta} \right. \\ &\quad \left. + \left(-n^\mu F_{\mu\alpha} e^{-8\phi/3} + 2(c_3 + \phi c_4 + 2c_5 (A_\gamma A^\gamma)^2) A_\alpha \right) \delta A^\alpha \right. \\ &\quad \left. + \left(-\frac{8}{3} n^\mu \partial_\mu \phi + (c_1 + 2\phi c_2 + A_\alpha A^\alpha c_4) \right) \delta\phi \right] \end{aligned} \tag{5.5}$$

where n^μ is the unit normal to the boundary, and $\pi_{\alpha\beta} = K_{\alpha\beta} - h_{\alpha\beta}K$.

We need to establish an appropriate class of variations. To do so, we need to define the phase space of solutions: that is, we need to specify the asymptotic boundary conditions on the fields. We choose our asymptotic boundary conditions to require that the leading falloff agrees with the γ^2 independent terms in eqs. (5.1), (5.2), (5.3). This requirement implies that no variation of these terms is allowed; these terms are non-dynamical. We further require that the subleading terms in the asymptotic falloff be related as in eqs. (5.1), (5.2), (5.3). This restrictive but permissible choice of boundary conditions by construction admits our black hole as an allowed solution. More precisely, the leading non-zero variation is required to be of the form

$$\delta h_{ab} = \frac{dh_{ab}}{d\gamma^2} \delta a \quad , \quad \delta A_a = \frac{dA_a}{d\gamma^2} \delta a \quad , \quad \delta \phi = \frac{d\phi}{d\gamma^2} \delta a \quad , \quad (5.6)$$

rather than allowing independent variations of the different fields at this order. We will denote this variation of the fields collectively by $\delta\psi_d$. In addition to this variation, we allow arbitrary variations $\delta\psi_f$ where the variations of δh_{ab} , δA_a and $\delta\phi$ are treated as independent and fall off at least one power of r^2 faster than in (5.6). Given that these $\delta\psi_f$ are independent, instead of considering the full variation (5.6), we are free to take a linear combination of $\delta\psi_d$ and the $\delta\psi_f$ and replace $\delta\psi_d$ with its leading order behavior:

$$\begin{aligned} \delta\psi_d : \quad \delta h^{uu} &= -\frac{1}{r^6} \delta a, & \delta h^{uv} &= \frac{1}{3r^4} \delta a, & \delta h^{vv} &= -\frac{1}{3r^2} \delta a, \\ \delta\phi &= -\frac{1}{2r^2} \delta a, & \delta A_u &= -\delta a, & \delta A_v &= -\frac{1}{r^2} \delta a. \end{aligned} \quad (5.7)$$

Substituting the asymptotic fields (5.1), (5.2), (5.3) into the general variation of the action (5.5), we find that the terms in $\delta\mathcal{S}$ which are independent of γ^2 are

$$\begin{aligned} \delta\mathcal{S}|_{\gamma^2=0} &= \frac{1}{16\pi G_5} \int d^4\xi \sqrt{-h} \left\{ r^4 (2 + c_3 - c_0) \delta h^{uu} - 2r^2 (c_0 - 3) \delta h^{uv} \right. \\ &\quad \left. + r^2 (c_0 - 3) (\delta h^{x_1 x_1} + \delta h^{x_2 x_2}) + c_1 \delta\phi + 2r^2 (c_3 - 1) \delta A^u \right\}. \end{aligned} \quad (5.8)$$

This part of the variation of the action will receive contributions that diverge like r^2 from $\delta\psi_d$, and finite contributions from $\delta\psi_f$. Since the variations of the different fields in $\delta\psi_f$ are independent, we need to set $c_0 = 3$, $c_1 = 0$, and $c_3 = 1$ to cancel these variations.

We are then left with contributions to $\delta\mathcal{S}$ which go like γ^2 ,

$$\begin{aligned} \delta\mathcal{S} &= \frac{\gamma^2}{16\pi G_5} \int d^4\xi \sqrt{-h} \left[-\left(\frac{13}{6} + \frac{c_4}{2} - 2c_5\right) r^2 \delta h^{uu} - 4\delta h^{uv} - \frac{2}{r^2} \delta h^{vv} \right. \\ &\quad \left. + \left(c_4 - c_2 - \frac{8}{3}\right) \frac{\delta\phi}{r^2} - \left(\frac{13}{3} + c_4 - 4c_5\right) \delta A^u - \frac{4}{r^2} \delta A^v \right]. \end{aligned} \quad (5.9)$$

These terms will receive finite contributions from the variations $\delta\psi_d$, and vanishing contributions from the variations $\delta\psi_f$. Using (5.7), the total variation of the action (5.9) becomes

$$\delta\mathcal{S} = \frac{\gamma^2}{16\pi G_5} \int d^4\xi \sqrt{-h} \frac{c_2 - 2c_4 + 4c_5 - 3}{2r^4} \delta a, \quad (5.10)$$

which will have the desired vanishing value provided $c_2 - 2c_4 + 4c_5 = 3$.

To summarize, an action which satisfies $\delta\mathcal{S} = 0$ for the restricted class of variations whose leading behaviour is given by (5.7) is

$$\begin{aligned} \mathcal{S} = & \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R - \frac{4}{3}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{4}e^{-8\phi/3}F_{\mu\nu}F^{\mu\nu} - 4A_\mu A^\mu - V(\phi) \right) \\ & + \frac{1}{16\pi G_5} \int d^4\xi \sqrt{-h} \left(2K - 6 + A_\mu A^\mu + c_4 A_\mu A^\mu \phi + c_5 (A_\mu A^\mu)^2 \right. \\ & \left. + (2c_4 - 4c_5 + 3)\phi^2 \right) \end{aligned} \quad (5.11)$$

for some arbitrary constants c_4 and c_5 .

5.3 Euclidean action for the black hole

Given the action (5.11) we can compute its value on the black hole solution (3.11), (3.12). We find that the on-shell value of the action is rather simple,

$$\mathcal{S} = \frac{1}{16\pi G_5} \int d^4\xi r_+^4. \quad (5.12)$$

In fact, it is identical to the on-shell action of the Schwarzschild-AdS₅ black hole (3.6)! Note that the dependence on c_4 and c_5 cancels out of the on-shell action, as does the dependence on γ^2 .

Let's now use this action to compute the thermodynamics of the black hole in the Euclidean approach. We will assume we compactify v with period Δv and the spatial directions with a volume V , and obtain a ‘‘Euclidean’’ solution by analytically continuing $t \rightarrow i\tau$. Smoothness of this analytically continued solution then forces the Euclidean time to have period $\Delta\tau = \pi/r_+$, consistent with our identification of the temperature (4.4). Thus, the full Euclidean action is

$$I = -\frac{\beta}{16G_5} r_+^3 V \Delta v, \quad (5.13)$$

where we have used the fact that $\beta\Delta v = \Delta y$ at fixed u .

We want to interpret this action as the saddle-point approximation to the grand canonical partition function,

$$\Xi(T, \mu) = e^{-\mathcal{Q}(T, \mu)/T} = \text{Tr} \left(\exp \left(-\frac{\hat{H}}{T} - \frac{\mu \hat{P}_v}{T} \right) \right) \approx e^{-I}, \quad (5.14)$$

with temperature T and chemical potential μ given as in (4.4) and (4.5) respectively. Note that the Euclidean action (5.13) is always negative, so the black hole solution makes the dominant contribution to this partition function for any non-zero temperature.

Thus, we should be able to extract the expected energy and charge as

$$\langle P_v \rangle = -T \frac{\partial}{\partial \mu} \ln \Xi(T, \mu) = T \frac{\partial}{\partial \mu} I, \quad (5.15)$$

$$\langle E \rangle + \mu \langle P_v \rangle = T^2 \frac{\partial}{\partial T} \ln \Xi(T, \mu) = -T^2 \frac{\partial}{\partial T} I. \quad (5.16)$$

Furthermore, using $\ln \Xi = -\mathcal{Q}/T$, we should have $\mathcal{Q}/T = (E + \mu P_v)/T - S = I$. So the entropy should be given by

$$S = - \left(T \frac{\partial}{\partial T} + 1 \right) I. \tag{5.17}$$

The action written in terms of T and μ is

$$I = - \frac{\pi^3 T^3}{64 G_5 \mu^2} V \Delta v, \tag{5.18}$$

which leads to

$$S = \frac{\pi^3 T^3}{16 G_5 \mu^2} V \Delta v, \tag{5.19}$$

which is the same as the result (4.1) we obtained earlier by direct calculation.¹¹

We then obtain the conserved charges

$$\langle N \rangle = \langle P_v \rangle \frac{\Delta v}{2\pi} = \frac{\pi^2 T^4}{64 G_5 \mu^3} V \Delta v^2, \tag{5.20}$$

and

$$\langle E \rangle = \frac{\pi^3 T^4}{64 G_5 \mu^2} V \Delta v. \tag{5.21}$$

Furthermore, the pressure is given in the grand canonical ensemble directly in terms of the Gibbs potential $\mathcal{Q}(T, \mu, V)$:

$$P V = -\mathcal{Q}(T, \mu, V) = \frac{\pi^3 T^4}{64 G_5 \mu^2} V \Delta v, \tag{5.22}$$

leading thus to an equation of state

$$P V = E. \tag{5.23}$$

A non-relativistic system with Galilean conformal invariance has different scalings for temporal and spatial directions as given in (2.2) for $\nu = 1$. This feature leads to an equation of state $d P V = 2 E$ in d -spatial dimensions [17], which is satisfied by (5.23). So indeed, the black hole solution constructed describes a state in the grand-canonical ensemble at temperature T and chemical potential μ for a non-relativistic conformal field theory.

The black hole solution (3.11) has a translationally invariant horizon in the field theory directions \mathbf{x} and hence in analogy with $\mathcal{N} = 4$ thermodynamics one expects that it corresponds to the high temperature phase of the Galilean CFT. Indeed this expectation is consistent with the fact that our free energy is always negative. In section 7 we discuss the possibility of a Hawking-Page like low temperature phase transition in finite volume.

¹¹That S is given both in terms of the horizon area and by (5.17) is a consistency check of our calculation: in general, by foliating the region outside the horizon by surfaces of constant time, we can always rewrite the Euclidean action as $I = \frac{1}{T}(E + \mu N) - S$, which implies the assumed relation between entropy and action.

6. Shear viscosity of non-relativistic plasmas

Strongly coupled non-relativistic plasmas that are encountered in cold atom systems, i.e. fermions at unitarity, are believed to behave as nearly ideal fluids [8, 9], like the quark-gluon plasma. Given that we have a holographic dual which describes the physics of a strongly coupled non-relativistic system, it is worth inquiring whether the shear viscosity of the plasma takes the universal value $\eta/s = 1/4\pi$ typical of such holographic systems [30].¹² In fact, given that the field theories we consider are similar to non-commutative Yang-Mills theories where it is known that $\eta/s = 1/4\pi$ [42], it is not surprising that we recover this same value, as we now show.¹³

To compute the shear viscosity, we will use the Kubo formula. Consider the following off-diagonal component of the Fourier transformed, retarded, two point function of the stress-tensor:¹⁴

$$G_{12,12}(\omega, 0) = -i \int du d^2x e^{i\omega u} \theta(u) \langle [T_{x_1x_2}(u, \mathbf{x}), T_{x_1x_2}(0, \mathbf{0})] \rangle. \quad (6.1)$$

The shear viscosity is given by the zero-frequency limit of this two point function,

$$\eta = - \lim_{\omega \rightarrow 0} \frac{1}{\omega} \text{Im} (G_{12,12}(\omega)). \quad (6.2)$$

To compute the shear viscosity, we use the recipes of refs. [17, 18] to compute this shear component of the two point function of the stress-tensor in the black hole background (3.11). Generic linearized fluctuations between the fields in the action (3.13) involve coupling between the gravitational, vector and dilatonic degrees of freedom. Happily, for the stress-tensor two point function, the dual field in the bulk is the metric fluctuation $\delta g_{x_1x_2}$ which decouples from the rest of the fluctuations at linear order. In fact, it turns out that $\delta g_{x_1}^{x_2}$ satisfies a massless, minimally coupled scalar equation in the background (3.11).

Consider then fluctuations of the bulk metric (3.11) in the spatial directions of the non-relativistic field theory. To obtain the shear viscosity we only need to know the zero momentum $\mathbf{p} = 0$ value of the correlator. Decomposing the fluctuation $\delta g_{x_1}^{x_2}$ into Fourier components and setting $\mathbf{p} = 0$,

$$\delta g_{x_1}^{x_2} \equiv e^{-i\omega u + i n v} \chi(\omega, r), \quad (6.3)$$

we have

$$\frac{f(r)}{r^5} \frac{d}{dr} \left(r^5 f(r) \frac{d\chi}{dr} \right) + V_{\text{eff}}(r) \chi(\omega, r) = 0, \quad (6.4)$$

¹²The bulk viscosity of the non-relativistic conformal plasmas vanishes due to the scale invariance [41, 7].

¹³The viscosity result was first presented by Adams at the BIRS workshop, who emphasized that the off-diagonal metric component behaves as a minimally coupled scalar and that the stress-tensor was dual to a mode with zero v -momentum.

¹⁴We will focus below mostly on the spatial components of the stress-tensor, which are the only tensorial objects in a Galilean field theory. The energy density T_{uu} and mass current T_{ui} can be incorporated if we work with a generalized stress-tensor complex.

where

$$V_{\text{eff}}(r) = \frac{1}{r^4} \left((1-f)\beta^2\omega^2 - (1+f)\omega n + \left(\frac{1}{4}(1-f) - \beta^2 r^2 f \right) \frac{n^2}{\beta^2} \right). \quad (6.5)$$

We have written the above expressions for general values of the v -momentum of the mode, which we call n . However, the stress tensor of the non-relativistic CFT must correspond to the $n = 0$ mode of the bulk metric. This can be seen in two different ways: first, as we argued previously, v -momentum corresponds to particle number in the non-relativistic CFT, and the stress tensor does not carry particle number. Second, the conformal dimension of the operators in the non-relativistic CFT will explicitly contain n dependence; for a massless minimally coupled bulk scalar field one has [17, 18]

$$\Delta = 2 + \sqrt{n^2 + 4}. \quad (6.6)$$

Such a scalar will have the same conformal dimension as the stress tensor only for $n = 0$. (The conformal dimension of a Galilean CFT in d spatial dimensions is $d + 2$.)

For $n = 0$, we find that (6.4) simplifies considerably. In fact, it becomes identical to the wave-equation for a massless scalar field at zero momentum on the Schwarzschild-AdS₅ black hole up to a trivial rescaling of the frequency by β . To calculate the Green's function, we solve as usual by demanding ingoing boundary conditions at the horizon, in the hydrodynamic limit $\omega/r_+ \ll 1$. Defining $\zeta(r)$ such that

$$\chi(\omega, r) \equiv A (r - r_+)^{-\frac{i\beta\omega}{4r_+}} \zeta(r), \quad (6.7)$$

where the constant A is related to the boundary value of $\chi(\omega, r)$, $\chi_0(\omega) = \chi(\omega, \infty)$. Solving for $\zeta(r)$ perturbatively in ω , we find that

$$\zeta(r) = 1 - \frac{i\beta\omega}{4r_+} \ln\left(\frac{f}{r - r_+}\right) + \mathcal{O}((\omega/r_+)^2). \quad (6.8)$$

The two-point function is then determined by the boundary term in the action for χ

$$\mathcal{S}_\chi = - \lim_{r \rightarrow \infty} \frac{1}{16\pi G_5} \int \frac{d\omega}{2\pi} dv d^2\mathbf{x} \frac{1}{2} r^5 f(r) \chi(-\omega, r) \frac{d\chi(\omega, r)}{dr} + \dots \quad (6.9)$$

We follow the recipe presented in ref. [43]. (See ref. [44] for a more rigorous treatment.) Using eqs. (6.7) and (6.8), the boundary term can be written

$$\mathcal{S}_\chi = \frac{1}{16\pi G_5} \int \frac{d\omega}{2\pi} dv d^2\mathbf{x} \chi_0(-\omega) \left(\frac{i}{2} r_+^3 \beta \omega \right) \chi_0(\omega) \quad (6.10)$$

from which the hydrodynamic retarded Green's function may be extracted:

$$G_{12,12}(\omega) = -\frac{i}{16\pi G_5} \beta \omega r_+^3 \Delta v. \quad (6.11)$$

The Kubo formula for the viscosity then leads to

$$\eta = \frac{1}{16\pi G_5} \beta r_+^3 \Delta v \quad (6.12)$$

which, noting that $\beta\Delta v = \Delta y$ at fixed u , gives

$$\frac{\eta}{s} = \frac{1}{4\pi}. \tag{6.13}$$

The result is in large part a consequence of the fact that the zero-frequency limit of the two-point function has a trivial β dependence, as does the entropy density obtained from (4.1).

7. Discussion

We have discussed aspects of holography for non-relativistic conformal field theories, concentrating in particular on the nonzero temperature physics of these systems. We described how non-relativistic field theories arise naturally in string theory from the world-volume theories on D-branes. The specific construction we focused on is DLCQ quantization of an R-charged twisted D3-brane world volume theory ($\mathcal{N} = 4$ SYM). However, the construction via the Null Melvin Twist makes it clear that one can generate a whole host of such theories with and without conformal invariance.

To generate non-relativistic conformal field theories in $d = 2$ spatial dimensions with dual geometries of the form (2.1) with $\nu = 1$, one can start with any $\mathcal{N} = 1$ superconformal field theory with an AdS dual. The infinite class of $\mathcal{N} = 1$ quiver gauge theories with $\text{AdS}_5 \times X_5$ duals, where X_5 is a Sasaki-Einstein manifold [45] (a special case of which is the Klebanov-Witten conifold theory [46]) provide excellent starting points for such constructions. In the dual spacetime one has a $U(1)_R$ realized as an isometry in X_5 analogous to the \mathbf{S}^1 fibration over \mathbf{CP}^2 used here. The Null Melvin Twist along this isometry will yield the appropriate pp-wave geometries, and what we have discussed in the text can be applied to Sasaki-Einstein spaces with little or no modification. It is easy to see that the non-relativistic equation of state (5.23) will be respected by these field theories. Similarly, one could start with relativistic field theories which have a non-trivial RG flow and construct non-conformal analogs of Galilean field theories. All these examples provide 2-spatial dimensional non-relativistic field theories.

It should be possible to generate solutions in higher dimensions $d > 2$ using other D-brane world-volume theories, but these would typically not exhibit Galilean conformal symmetry. To obtain a Galilean CFT in $d = 3$ spatial dimensions, we would need to start with an asymptotic AdS_6 geometry, which would correspond to the strongly coupled fixed point of a five dimensional CFT. Other generalizations of course include the M-brane world-volume CFTs, wherein the asymptotic AdS_4 and AdS_7 spacetimes can be twisted with the R-symmetries, which is geometrically achieved by turning on 3-form fluxes in the background.

Even without the precise non-extremal geometries at hand, one can infer some thermodynamic features of the higher dimensional relativistic CFTs, if we assume that as in the case studied here, the gravitational and matter degrees of freedom conspire to give a Euclidean action which agrees with the action for the untwisted asymptotically AdS black hole. In d -spatial dimensions we would then have (gathering all the irrelevant numerical

coefficients in Γ and Γ')

$$I = -\Gamma' \beta r_+^{d+1} = -\Gamma \frac{T^{d+1}}{\mu^{\frac{d}{2}+1}} . \quad (7.1)$$

The scaling here follows from $T = \frac{(d+2)r_+}{4\pi\beta}$ and we expect μ is still given by (4.5). From here it is trivial to check that

$$E = \Gamma \frac{d}{2} \frac{T^{d+2}}{\mu^{\frac{d}{2}+1}} \implies E = \frac{d}{2} P. \quad (7.2)$$

Note that in converting the entropy and Gibbs potential into field theory quantities, because of the compact v direction, we introduce a chemical potential μ that plays no role in the untwisted backgrounds. For the reasons outlined in section 2 we believe that the appropriate holographic description of these systems is in terms of the asymptotic pp-wave spacetimes (2.1) even though the geometric evaluation of the Gibbs potential and the entropy of these non-relativistic CFTs, expressed as a function of the horizon radius r_+ , leads to answers that are identical to those obtained for black holes in untwisted AdS_{d+3} .

The thermodynamics we have discussed is for non-relativistic CFTs in non-compact space. It would be interesting to study these systems in finite volume, and to see if they exhibit phase transitions like the Hawking-Page transition that is well known in the AdS case. We suspect the answer is yes given the similarities of the Euclidean action computation (5.12), and the fact that we obtain these theories by deforming relativistic gauge theories. However, if we start from $\mathcal{N} = 4$ SYM on a compact manifold, say $\mathbf{S}^3 \times \mathbf{R}$, one needs to pick an appropriate light-cone to carry out the deformation. For the case of \mathbf{S}^3 one could presumably use the non-degenerate Hopf fibre direction to define appropriate light-cone coordinates and twist the theory by the R-symmetry.

Apart from the intrinsic interest in developing gravity duals to non-relativistic condensed matter systems, these Galilean CFTs may further the recent developments in the fluid-gravity correspondence [47]. Any interacting field theory in an appropriate long-wavelength limit can be modeled as a hydrodynamic system; recently ref. [47] developed a precise dictionary between the dynamics of relativistic conformal fluids and asymptotically AdS black hole solutions. While this construction provides an interesting avenue to explore the physics of fluid dynamics in a holographic setting, our knowledge of relativistic fluids is less well developed than that of non-relativistic fluids. There should be more ways of cross-checking holographic duals of strongly coupled non-relativistic field theories.

The R-charge twisted $\mathcal{N} = 4$ theory discussed in the paper provides in the hydrodynamic limit an example of a two dimensional fluid, where hydrodynamic features such as turbulence differ qualitatively from their higher dimensional counter-parts owing to the inverse cascade phenomenon [48]. In the context of the fluid-gravity correspondence, ref. [49] suggested that one might see qualitative differences between gravity in different dimensions, based on qualitative differences in the turbulent regime. Precisely because most work on turbulence is done for non-relativistic fluids, the non-relativistic system studied here may provide a particularly good playground for exploring turbulence. It should however be

borne in mind that non-relativistic conformal fluids are also highly compressible,¹⁵ owing to the equation of state (5.23), which is a consequence of scale invariance. The models which violate Galilean conformal invariance would thus be better starting points to realize incompressible fluids.

Acknowledgments

It is a pleasure to thank Allan Adams, Rajesh Gopakumar, Sean Hartnoll, Veronika Hubeny, Pavel Kovtun, Don Marolf, John McGreevy, Shiraz Minwalla, Dam Son, Andrei Starinets and Larry Yaffe for very interesting discussions. We would also like to thank Veronika Hubeny and Larry Yaffe for comments on the manuscript. CH and MR would like to thank the Galileo Galilei Institute for Theoretical Physics, Firenze for hospitality during the workshop “Non-Perturbative Methods in Strongly Coupled Gauge Theories” and the INFN for partial support. MR and SFR would like to thank the ICMS, Edinburgh for hospitality during the workshop “Gravitational Thermodynamics and the Quantum Nature of Spacetime”. CPH would like to thank the BIRS for hospitality during the workshop “Emerging Directions in String Theory”. MR would also like to thank the Tata Institute of Fundamental Research, Mumbai, for wonderful hospitality during the “Monsoon Workshop on String Theory” and would like to thank the International Center for Theoretical Sciences, India for hospitality and support. MR and SFR are partially supported by STFC. CPH was supported in part by the NSF under Grant No. PHY-0756966.

A. Anisotropic Galilean field theories

The discussion in the text has been confined to spacetimes of the form (2.1) with $\nu = 1$ supported by matter that preserves the rotational invariance in the spatial directions. However, it is just as easy to construct spacetimes where the rotational invariance is broken by a simple generalization of the construction in [17]. We would like to obtain the metric (related to (2.1) by $r = 1/z$)

$$ds^2 = \frac{1}{z^2} (-2 du dv + d\mathbf{x}_d + dz^2) - \frac{1}{z^{2\nu+2}} du^2 \quad (\text{A.1})$$

in d -spatial dimensions as the solution for some equations of motion with some appropriate matter. The matter stress tensor supporting the solution is

$$T_{uu} \propto \frac{1}{z^{2\nu+2}}. \quad (\text{A.2})$$

In [17] this stress tensor was modeled by a massive vector. However, we can do just as well with a pair of p -form fields. Let us consider the action

$$\mathcal{S} = \int d^{d+3}x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{2} |H_{(p+1)}|^2 - \frac{1}{2} |F_{(d+3-p)}|^2 \right) - \nu \int B_{(p)} \wedge F_{(d+3-p)}, \quad (\text{A.3})$$

¹⁵It may however be possible to focus on the low lying shear mode to obtain incompressible Navier-Stokes flow. We thank Dam Son for alerting us to this possibility.

with $H_{(p+1)} = dB_{(p)}$ and $F_{(d+3-p)} = dC_{(d+2-p)}$. An appropriate source is

$$\begin{aligned} H_{(p+1)} &= -\frac{\alpha(\nu+p)}{z^{\nu+p+1}} dz \wedge du \wedge \omega_{(p-1)}, \\ F_{(d+3-p)} &= \frac{\alpha(\nu+p)}{z^{\nu+d+3-p}} dz \wedge du \wedge (\star\omega)_{(d-p+1)}, \end{aligned} \quad (\text{A.4})$$

where ω_p is an arbitrary p -form on the spatial \mathbf{R}^d (parameterized by \mathbf{x}_d) and $(\star\omega)_{(d-p)}$ is its Hodge dual. It is easy to check that this ansatz satisfies the field equations coming from (A.3) and provides the appropriate stress tensor (A.2). In fact the massive vector theory of [17] corresponds to choosing the $\omega_{(p-1)}$ on the spatial sections that appears in the H -flux to be a scalar and hence the dual form to be the top-form on \mathbf{R}^d , which naturally gives a mass term (like in massive Type IIA string theory). In more general cases, the presence of the fluxes breaks rotational invariance — while the metric (A.1) has full $\text{SO}(d+1)$ rotational symmetry, the fluxes break $\text{SO}(d+1)$ down to $\text{SO}(p) \times \text{SO}(d+2-p)$.

This construction is in fact inspired by the Null Melvin Twist along the world-volume directions of the D-brane to obtain light-like non-commutative Yang-Mills theories [27]. Starting from the non-extremal D3-brane solution (3.6), we find a solution to the IIB equations of motion (for simplicity twisting only along x_1 — for a more general twist see the solution given in Eq (C.6) of ref. [28])

$$ds_{\text{str}}^2 = e^{2\varphi} \left[r^2 \left(-f(r) dt^2 + dy^2 + dx_1^2 \right) - \beta^2 f(r) r^6 (dt + dy)^2 \right] + r^2 dx_2^2 + \frac{dr^2}{r^2 f(r)} + d\Omega_5^2, \quad (\text{A.5})$$

supported by NS-NS ($B_{(2)}$) and RR ($C_{(2)}$ and $C_{(4)}$) fluxes in 10 dimensional Type IIB supergravity,

$$\begin{aligned} B_{(2)} &= \beta r^4 e^{2\varphi} (dt + dy) \wedge dx_1, \\ C_{(2)} &= -\beta r^4 (dt + dy) \wedge dx_2, \\ F_{(5)} &= 4(1 + \star) \text{Vol}(\mathbf{S}^5), \\ e^\phi &= \frac{1}{\sqrt{1 + \beta^2 r_+^4}}, \\ f(r) &= 1 - \frac{r_+^4}{r^4}. \end{aligned} \quad (\text{A.6})$$

In the geometry (A.5) we have $\nu = 2$ due to the fact that $g_{uu} \sim r^6$ and the dilaton is constant. Furthermore, the \mathbf{S}^5 is also undeformed, unlike the situation encountered in (3.7). Reducing (A.5) on the \mathbf{S}^5 we find an effective 5-dimensional action of the general form (A.3)

$$\begin{aligned} \mathcal{S}_{5\text{def}} &= \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[e^{-2\varphi} \left(R - 2\Lambda - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right) - \frac{1}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda} \right] \\ &\quad + \frac{1}{4\pi G_5} \int B_2 \wedge F_3. \end{aligned} \quad (\text{A.7})$$

The five dimensional solution is then given by the fields

$$\begin{aligned}
 ds_{\text{str}}^2 &= e^{2\varphi} \left[r^2 (-f(r) dt^2 + dy^2 + dx_1^2) - \beta^2 f(r) r^6 (dt + dy)^2 \right] + r^2 dx_2^2 + \frac{dr^2}{r^2 f(r)}, \\
 B_{(2)} &= \beta r^4 e^{2\varphi} (dt + dy) \wedge dx_1, \\
 C_{(2)} &= -\beta r^4 (dt + dy) \wedge dx_2, \\
 e^\varphi &= \frac{1}{\sqrt{1 + r_+^4 \beta^2}},
 \end{aligned} \tag{A.8}$$

for a choice of the cosmological constant

$$\Lambda = -6 - 4r_+^4 \beta^2 e^{2\varphi} \tag{A.9}$$

which depends non-trivially on the deformation parameter $\gamma^2 = \beta^2 r_+^4$. This solution is the gravitational dual to light-like non-commutative $\mathcal{N} = 4$ super Yang-Mills and it would be interesting to study this example further to understand aspects of non-relativistic dynamics.

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